MATH 1650 SOLVING EQUATIONS INVOLVING EXPONENTIAL FUNCTIONS

1.

$$\begin{aligned}
2^{3x} &= 16^{1-x} \\
2^{3x} &= \left(2^4\right)^{1-x} && \text{Since } 16 = 2^4. \\
2^{3x} &= 2^{4(1-x)} && \text{Properties of exponents.} \\
3x &= 4(1-x) && \text{One-to-one property of exponential functions.} \\
3x &= 4 - 4x \\
7x &= 4 \\
x &= \frac{4}{7}
\end{aligned}$$

2.

$$\begin{aligned}
2000 &= 1000 \cdot 3^{-0.1t} \\
2 &= 3^{-0.1t} && \text{Divide both sides by 1000.} \\
\ln(2) &= \ln\left(3^{-0.1t}\right) && \text{Take natural logs.} \\
\ln(2) &= -0.1t\ln(3) && \text{Power Rule of Logs.} \\
\frac{\ln(2)}{\ln(3)} &= -0.1t && \text{Divide both sides by } \ln(3). \\
-\frac{10\ln(2)}{\ln(3)} &= t && \text{Divide both sides by } -0.1 \text{ (so multiply both sides by } -10.)
\end{aligned}$$

3.

$$\begin{aligned}
9 \cdot 3^x &= 7^{2x} \\
3^2 \cdot 3^x &= 7^{2x} && \text{Rewrite } 9 = 3^2 \\
3^{x+2} &= 7^{2x} && \text{Product Rule of Exponents.} \\
\ln\left(3^{x+2}\right) &= \ln\left(7^{2x}\right) && \text{Take natural logs.} \\
(x+2)\ln(3) &= 2x\ln(7) && \text{Power Rule of Logs.} \\
x\ln(3) + 2\ln(3) &= 2x\ln(7) && \text{Distribute.} \\
2\ln(3) &= 2x\ln(7) - x\ln(3) && \text{Subtract } x\ln(3) \text{ from both sides.} \\
2\ln(3) &= x(2\ln(7) - \ln(3)) && \text{Factor.} \\
\frac{2\ln(3)}{2\ln(7) - \ln(3)} &= x && \text{Divide both sides by } (2\ln(7) - \ln(3)).
\end{aligned}$$

4.

$$
\begin{aligned}
75 &= \frac{100}{1+3e^{-2t}} \\
75\left(1+3e^{-2t}\right) &= 100 && \text{Multiply both sides by } 1+3e^{-2t} \\
75+225e^{-2t} &= 100 && \text{Distribute.} \\
225e^{-2t} &= 25 && \text{Subtract 75 from both sides.} \\
e^{-2t} &= \frac{1}{9} && \text{Divide both sides by 225.} \\
\ln\left(e^{-2t}\right) &= \ln\left(\frac{1}{9}\right) && \text{Take natural logs.} \\
-2t &= \ln\left(\frac{1}{9}\right) && \text{Inverse property of } \ln(x) \text{ and } e^x. \\
-2t &= -\ln(9) && \text{Property of Logs: } \ln\left(\frac{1}{9}\right) = \ln\left(9^{-1}\right) = -\ln(9) \\
t &= \tfrac{1}{2}\ln(9) && \text{Divide both sides by 2.} \\
t &= \ln\left(9^{\frac{1}{2}}\right) = \ln(3) && \text{Power Rule of Logs.}
\end{aligned}
$$

1. Rewriting $25 = 5^2$, we get $\left(5^2\right)^x = 5^x + 6$, or $5^{2x} = 5^x + 6$.

   Rearranging, we have $5^{2x} - 5^x - 6 = 0$, which is a quadratic in disguise. (Do you see why?)

   Factoring gives: $(5^x - 3)(5^x + 2) = 0$. Setting each factor equal to 0 gives $5^x - 3 = 0$ or $5^x + 2 = 0$.

   From $5^x - 3 = 0$, we get $5^x = 3$. Taking natural logs gives $\ln(5^x) = \ln(3)$ or $x\ln(5) = \ln(3)$.

   Hence, one answer is: $x = \frac{\ln(3)}{\ln(5)}$.

   The equation $5^x + 2 = 0$ reduces to $5^x = -2$ which has no real solution (Do you see why?)

2. Clearing the denominator in $\frac{e^x - e^{-x}}{2} = 5$ gives $e^x - e^{-x} = 10$.

   Rewriting $e^{-x} = \frac{1}{e^x}$, we see we have another denominator to clear: $e^x - \frac{1}{e^x} = 10$.

   Doing so gives $e^{2x} - 1 = 10e^x$ or $e^{2x} - 10e^x - 1 = 0$. This is yet another quadratic in disguise!

   Since this quadratic doesn't easily factor, we use the quadratic formula with $a = 1$, $b = -10$, and $c = -1$.

   We get $e^x = 5 \pm \sqrt{26}$. Since $5 - \sqrt{26} < 0$, we get no real solution to $e^x = 5 - \sqrt{26}$. (Why not?)

   For $e^x = 5 + \sqrt{26}$, we take natural logs to obtain $x = \ln\left(5 + \sqrt{26}\right)$.